

# Density Evolution in the New Modified Chaplygin Gas Model

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In this paper, we have considered new modified Chaplygin gas (NMCG) model which interpolates between radiation at early stage and  $\Lambda$ CDM at late stage. This model is regarded as a unification of dark energy and dark matter (with general form of matter). We have derived the density parameters from the equation of motion for the interaction between dark energy and dark matter. Also we have studied the evolution of the various components of density parameters.

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## I. INTRODUCTION

The standard cosmological model (SCM) can only describe decelerated universe models and so cannot reproduce the results coming from the recent type Ia supernovae observations upto about  $z \sim 1$  [1] which favour an accelerated current universe. But the SCM can give a satisfactory explanation to other observational properties of the present universe. The recent extensive search for a matter field has given rise to the concept of an accelerated expansion for the universe. This type of matter is called Q-matter. This Q-matter can behave like a cosmological constant [2] by combining +ve energy density and negative pressure. So there must be this Q-matter either neglected or unknown responsible for this accelerated universe. At the present epoch, a lot of works has been done to solve this quintessence problem and most popular candidates for Q-matter has so far been a scalar field having a potential which generates a sufficient negative pressure. Furthermore, observations reveal that the unknown form of matter properly referred to as the ‘dark energy’ accounts for almost 70% of the universe. This is confirmed by the very recent WMAP data [3]. A large number of possible candidates for this ‘dark energy’ component has already been proposed and their behaviour have been studied extensively [4]. In order to explain the nature of dark energy, many models have been proposed, such as, k-essence [5, 6], tachyon [7], phantom [8], spintessence [9], etc. So far, the theoretical probe of dark energy focuses mainly on the evolution of the dark energy density or the equation of state. The current astronomical observations data cannot determine completely the nature of dark energy [10].

Another alternative candidate for Q-matter is exotic type of fluid – the so-called Chaplygin gas which obeys the equation of state  $p = -\tilde{A}/\rho$ , ( $\tilde{A} > 0$ ) [11], where  $p$  and  $\rho$  are respectively the pressure and energy density. Subsequently, the above equation was generalized to the form  $p = -\tilde{A}/\rho^\alpha$ ,  $0 \leq \alpha \leq 1$  [12, 13] and recently it was modified to the form  $p = \gamma\rho - \tilde{A}/\rho^\alpha$ , ( $\gamma > 0$ ) [14, 15], which is known as *Modified Chaplygin Gas* (MCG). This model represents the evolution of the universe starting from the radiation era to the  $\Lambda$ CDM model. Recently Guo and Jhang [16] proposed variable Chaplygin gas model where  $\tilde{A}$  is a positive function of the cosmological scale factor ‘ $a$ ’ i.e.,  $\tilde{A} = \tilde{A}(a)$ . This assumption is reasonable since  $\tilde{A}(a)$  is related to the scalar potential if we take the Chaplygin gas Born-Infeld scalar field [17]. Plethora of literatures is available on the study of variable Chaplygin gas model [18]. Also, New Generalized Chaplygin Gas (NGCG) model have been discussed by Jhang et al [19].

Another type of dark energy roughly includes quiescence (or  $X$ -matter) [20]. The quiescence or  $X$ -matter component is simply characterized by a constant, non-positive equation of state  $w_x$ , where  $w_x$  is the ratio of pressure and density of this  $X$ -matter. For accelerating universe,  $w_x < -1/3$ . For a normal scalar field with potential in FRW background with the presence of cold dark matter (CDM), the bound of  $w_x$  would be  $-1 < w_x < 0$ . Also  $w_x < -1$  is possible in the framework of  $X$ CDM ( $X$ -matter with CDM) by fitting the SNe Ia data. From observational data, the range of the equation of state  $w_x$  of dark energy has been determined as  $-1.46 < w_x < -0.78$ . Now MCG generalizes to accommodate any possible  $X$ -type dark energy. So we consider here New Modified Chaplygin Gas (NMCG) model as a scheme for unification of  $X$ -type dark energy and dark matter. The new feature of this model is that it behaves as a dark matter (radiation) at early stage and  $X$ -type dark energy at late stage. We will show that this model is a kind of interacting  $X$ CDM system.

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The organization of the paper is as follows: In section II, we have considered New Modified Chaplygin Gas (NMCG) model and find out the energy densities for dark energy and dark matter of this model in FRW universe. Section III describes the interaction between dark energy and dark matter and the evolution of density parameters for dark energy and dark matter. Finally, the paper ends with concluding remarks in section IV.

## II. FRW MODEL AND NMCG MODEL

The metric of a homogeneous and isotropic universe in the FRW model is

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (1)$$

where  $a(t)$  is the scale factor and  $k$  ( $= 0, \pm 1$ ) is the curvature scalar. The Einstein field equations are

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3} \rho \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6} (\rho + 3p) \quad (3)$$

where  $\rho$  and  $p$  are energy density and isotropic pressure respectively (choosing  $8\pi G = c = 1$ ).

The equation of state for NMCG model is

$$p_{ch} = \gamma \rho_{ch} - \frac{\tilde{A}(a)}{\rho_{ch}^\alpha}, \quad \gamma > 0, \quad 0 \leq \alpha \leq 1 \quad (4)$$

where  $\tilde{A}(a)$  is a function that depends upon the scale factor of the universe.

In the framework of FRW cosmology, considering the exotic background fluid, the NMCG is described by the equation of state

$$\dot{\rho}_{ch} + 3\frac{\dot{a}}{a}(\rho_{ch} + p_{ch}) = 0 \quad (5)$$

We know that the exotic background fluid smoothly interpolates between a dark matter dominated phase  $\rho \sim a^{-3(1+\gamma)}$  to dark energy dominated phase  $\rho \sim a^{-3(1+w_X)}$  where  $w_X$  ( $< -1/3$ ) is the ratio of pressure and energy density of  $X$ -matter (dark energy). For this purpose, without any loss of generality, we consider the function  $\tilde{A}(a)$  has in the form [19]

$$\tilde{A}(a) = -w_X A a^{-3(1+w_X)(1+\alpha)}, \quad A > 0 \quad (6)$$

so that using equations (4)-(6), the energy density of the NMCG can be expressed as

$$\rho_{ch} = \left[ \frac{w_X}{w_X - \gamma} A a^{-3(1+w_X)(1+\alpha)} + B a^{-3(1+\gamma)(1+\alpha)} \right]^{\frac{1}{1+\alpha}} \quad (7)$$

Now NMCG scenario involves an interacting  $X$ CDM system. For showing this, we first decompose the NMCG fluid into two components i.e., dark energy and dark matter components (i.e.,  $\rho_{ch} = \rho_X + \rho_{dm}$ ). There are several works on such decomposition procedure in Chaplygin gas model [19, 21]. So we can obtained the densities of dark energy and dark matter components respectively as

$$\rho_X = \frac{\frac{w_X}{w_X - \gamma} A a^{-3(1+w_X)(1+\alpha)} + \frac{\gamma}{w_X} B a^{-3(1+\gamma)(1+\alpha)}}{\left[ \frac{w_X}{w_X - \gamma} A a^{-3(1+w_X)(1+\alpha)} + B a^{-3(1+\gamma)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}} \quad (8)$$

and

$$\rho_{dm} = \frac{\frac{w_X - \gamma}{w_X} B a^{-3(1+\gamma)(1+\alpha)}}{\left[ \frac{w_X}{w_X - \gamma} A a^{-3(1+w_X)(1+\alpha)} + B a^{-3(1+\gamma)(1+\alpha)} \right]^{\frac{1}{1+\alpha}}} \quad (9)$$

From these two expressions one obtains the scaling behaviour of the energy densities

$$\frac{\rho_{dm}}{\rho_X} = \frac{\frac{w_X - \gamma}{w_X} B a^{-3(1+\gamma)(1+\alpha)}}{\frac{w_X}{w_X - \gamma} A a^{-3(1+w_X)(1+\alpha)} + \frac{\gamma}{w_X} B a^{-3(1+\gamma)(1+\alpha)}} \quad (10)$$

Parameters  $A$  and  $B$  can be expressed using current cosmological observations. It is easy to get

$$A + B = \rho_{ch0}^\eta \quad (11)$$

where,  $\eta = 1 + \alpha$  is used to characterize the interaction for simplicity, thus we have

$$A = A_s \rho_{ch0}^\eta, \quad B = (1 - A_s) \rho_{ch0}^\eta \quad (12)$$

where  $A_s$  is a dimensional parameter. Using equations (10) and (12), one gets

$$A_s = \frac{\rho_{X0}(\gamma - w_X)^2 + \rho_{dm0}\gamma(\gamma - w_X)}{(w_X^2 - \gamma w_X + \gamma^2)\rho_{dm0} + (\gamma - w_X)^2\rho_{X0}} \quad (13)$$

Here, we have assumed that the universe is flat. Hence the NMCG energy density can be expressed as

$$\rho_{ch} = \rho_{ch0} a^{-3(1+\gamma)} \left[ 1 - A_s \left( 1 - \frac{w_X}{w_X - \gamma} a^{-3\eta(w_X - \gamma)} \right) \right]^{\frac{1}{\eta}} \quad (14)$$

Making use of (7), (8), (9) and (14), the energy densities of dark energy and dark matter can be re-expressed as

$$\rho_X = \rho_{X0} a^{-3(1+\gamma)} \frac{\frac{\gamma}{w_X} (1 - A_s) + \frac{w_X A_s}{w_X - \gamma} a^{-3\eta(w_X - \gamma)}}{\frac{\gamma}{w_X} (1 - A_s) + \frac{w_X A_s}{w_X - \gamma}} \left( 1 + \frac{\gamma A_s}{w_X - \gamma} \right)^{1 - \frac{1}{\eta}} \left[ 1 - A_s \left( 1 - \frac{w_X}{w_X - \gamma} a^{-3\eta(w_X - \gamma)} \right) \right]^{\frac{1}{\eta} - 1} \quad (15)$$

$$\rho_{dm} = \rho_{dm0} a^{-3(1+\gamma)} \left( 1 + \frac{\gamma A_s}{w_X - \gamma} \right)^{1 - \frac{1}{\eta}} \left[ 1 - A_s \left( 1 - \frac{w_X}{w_X - \gamma} a^{-3\eta(w_X - \gamma)} \right) \right]^{\frac{1}{\eta} - 1} \quad (16)$$

### III. INTERACTION BETWEEN DARK MATTER AND DARK ENERGY

The whole NMCG fluid satisfies the energy conservation, but dark energy and dark matter components do not obey the energy conservation separately; they interact with each other. We portray the interaction through an energy exchange term  $Q$ . The equations of motion for dark energy and dark matter can be written as

$$\dot{\rho}_X + 3H(1 + w_x)\rho_X = Q \quad (17)$$

$$\dot{\rho}_{dm} + 3H(1 + \gamma)\rho_{dm} = -Q \quad (18)$$

where,  $H = \frac{\dot{a}}{a}$  represents the Hubble parameter. We define the effective equations of state for dark energy and dark matter through the parameters

$$w_x^{(e)} = w_x - \frac{Q}{3H\rho_x} \quad (19)$$

$$w_{dm}^{(e)} = \gamma + \frac{Q}{3H\rho_{dm}} \quad (20)$$

The equations of dark energy and dark matter can be re-expressed as

$$\dot{\rho}_x + 3H(1 + w_x^{(e)})\rho_x = 0 \quad (21)$$

$$\dot{\rho}_{dm} + 3H(1 + w_{dm}^{(e)})\rho_{dm} = 0 \quad (22)$$

By means of equations (15), (16), (21), and (22) one can obtain

$$w_x^{(e)} = \gamma + w_x \left[ \frac{\frac{w_x A_s}{w_x - \gamma} + (1 - A_s) a^{3\eta(w_x - \gamma)} \left( (1 - \eta) \frac{\gamma}{w_x} + \eta \right)}{\left( \frac{\gamma(1 - A_s)}{w_x A_s} a^{3\eta(w_x - \gamma)} + \frac{w_x}{w_x - \gamma} \right) \left( (1 - A_s) a^{3\eta(w_x - \gamma)} + \frac{w_x A_s}{w_x - \gamma} \right)} \right] \quad (23)$$

$$w_{dm}^{(e)} = \gamma + \left[ \frac{(\eta - 1)w_x A_s}{(1 - A_s) a^{3\eta(w_x - \gamma)} + \frac{w_x A_s}{w_x - \gamma}} \right] \quad (24)$$

Considering the spatially flat universe, the Friedmann equation can be written as

$$3M_p^2 H^2 = \rho_{ch} + \rho_b \quad (25)$$

where,  $M_p$  is the reduced Plank mass and  $\rho_b$  is the baryon matter density. The Friedmann equation can also be expressed as

$$H(a) = H_0 E(a) \quad (26)$$

where

$$E(a) = \left[ (1 - \Omega_b^0) a^{-3} \left\{ (1 - A_s) a^{-3(1+\alpha)\gamma} + \left( \frac{w_x A_s}{w_x - \gamma} \right) a^{-3(1+\alpha)w_x} \right\}^{\frac{1}{1+\alpha}} + \Omega_b^0 a^{-3} \right]^{1/2} \quad (27)$$

Then the density parameters of various components can be obtained,

$$\Omega_X = \Omega_X^0 E^{-2} a^{-3(1+\gamma)} \frac{\frac{\gamma}{w_x} (1 - A_s) + \frac{w_x A_s}{w_x - \gamma} a^{-3\eta(w_x - \gamma)}}{\frac{\gamma}{w_x} (1 - A_s) + \frac{w_x A_s}{w_x - \gamma}} \left( 1 + \frac{\gamma A_s}{w_x - \gamma} \right)^{1 - \frac{1}{\eta}} \left[ 1 - A_s \left( 1 - \frac{w_x}{w_x - \gamma} a^{-3\eta(w_x - \gamma)} \right) \right]^{\frac{1}{\eta} - 1} \quad (28)$$

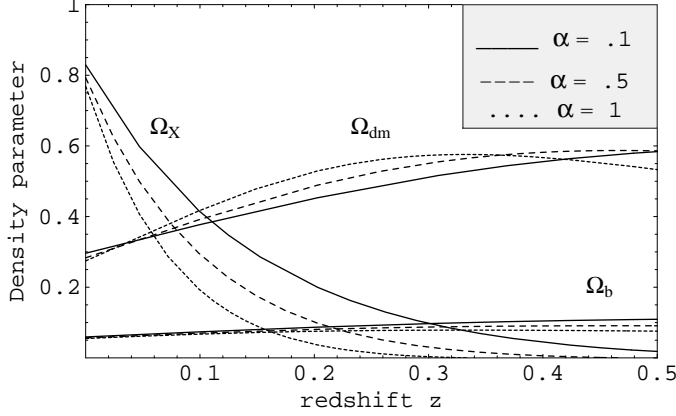


Fig.1

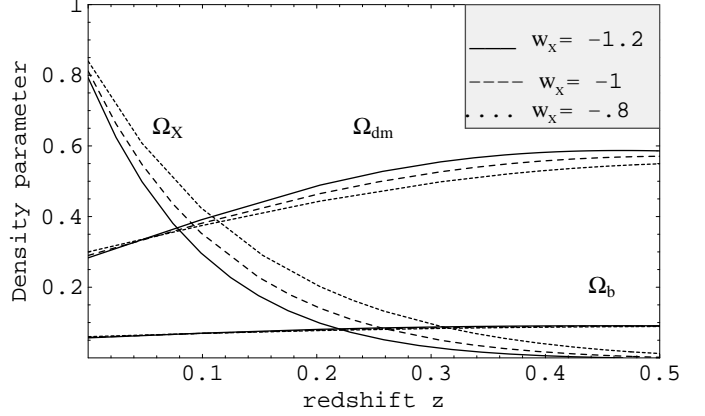


Fig.2

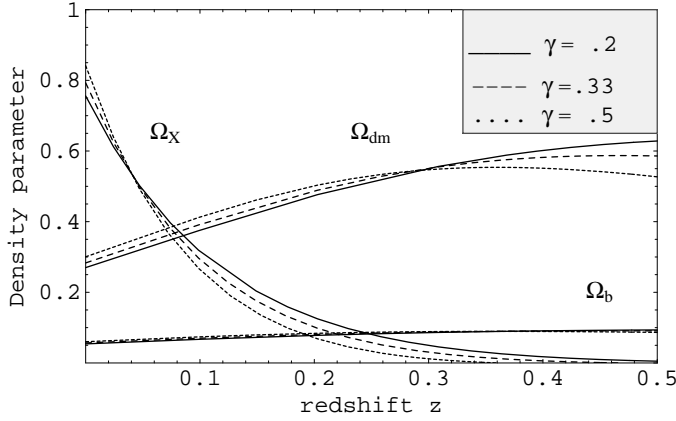


Fig.3

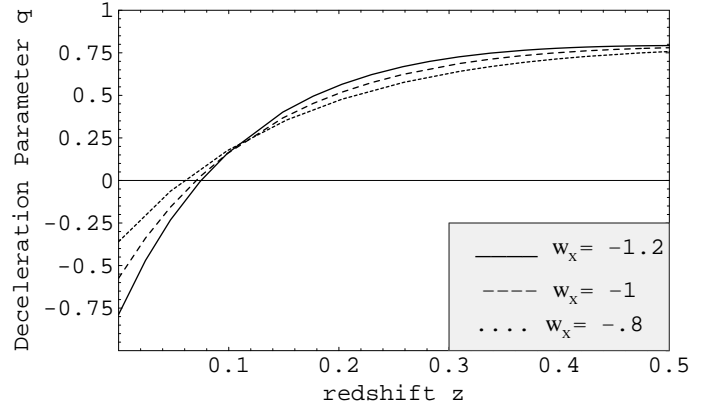


Fig.4

**Figs. 1 - 3** show the evolution of the density parameters for various components  $\Omega_X$ ,  $\Omega_{dm}$  and  $\Omega_b$ . The current density parameters used in the plots are  $\Omega_{ch}^0 = 0.25$ ,  $\Omega_X^0 = 0.7$  and  $\Omega_b^0 = 0.05$ . In this case, Fig. 1 shows  $w_X$  and  $\gamma$  fixed and  $\alpha$  is varied; Fig. 2 shows  $\alpha$  and  $\gamma$  fixed and  $w_X$  is varied; Fig. 3 shows  $\alpha$  and  $w_X$  fixed and  $\gamma$  is varied.

**Fig. 4** shows the evolution of the deceleration parameter  $q(z)$ . The current density parameters used in the plots are  $\Omega_{ch}^0 = 0.25$ ,  $\Omega_X^0 = 0.7$  and  $\Omega_b^0 = 0.05$ . In this case,  $w_X$  are taken to be  $-0.8$ ,  $-1$  and  $-1.2$ .

$$\Omega_{dm} = (1 - \Omega_X^0 - \Omega_b^0)E^{-2} a^{-3(1+\gamma)} \left(1 + \frac{\gamma A_s}{w_X - \gamma}\right)^{1-\frac{1}{\eta}} \left[1 - A_s \left(1 - \frac{w_X}{w_X - \gamma} a^{-3\eta(w_X - \gamma)}\right)\right]^{\frac{1}{\eta}-1} \quad (29)$$

$$\Omega_b = \Omega_b^0 E^{-2} a^{-3} \quad (30)$$

The density evolution of the NMCG model is given in Figures 1, 2 and 3. The current density parameters used in the plots are  $\Omega_{dm}^0 = 0.25$ ,  $\Omega_X^0 = 0.7$  and  $\Omega_b^0 = 0.05$ . In figure 1, we show the cases having the common equation-of-state parameters  $w_X = -1.2$  and  $\gamma = 1/3$ , while the parameter  $\alpha$  are taken to be 0.1, 0.5 and 1 respectively. In figure 2, the evolution of the density parameter is studied keeping the parameters  $\alpha$  fixed at 0.5 and  $\gamma = 1/3$  and the equation-of-state parameters  $w_X$  are taken to be  $-0.8$ ,  $-1$  and  $-1.2$ . Evolution of density with varying  $\gamma$  is depicted in figure 3. In this figure, the parameter  $\alpha$  is fixed at 0.5, equation-of-state parameter  $w_X$  is taken as  $-1.2$ , and values of  $\gamma$  are taken to be  $1/5$ ,  $1/3$ , and  $1/2$ . The acceleration of the Universe is evaluated by the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} \quad (31)$$

In the NMCG model, the deceleration parameter comes out to be

$$q = \frac{1}{2} (1 + 3w_x \Omega_X + 3\gamma \Omega_{dm}) \quad (32)$$

Evolution of the deceleration parameter  $q$  is shown in figure 4.

#### IV. CONCLUDING REMARKS

The new modified Chaplygin gas model is regarded as a unification of dark energy and dark matter (with general form of matter i.e.,  $\gamma \neq 0$ ). This model interpolates between radiation at early stage and  $\Lambda$ CDM at late stage. The unification of dark energy and dark matter should accommodate the quintessence-like ( $-1 < w_x < -1/3$ ) and phantom-like ( $w_x < -1$ ) dark energy. From figures 1 - 3, we have seen that  $\Omega_{dm}$  first increases and then decreases to a constant value and  $\Omega_X$  decreases to a constant value but in every stage in the evolution of the universe, the sum remains approximately equal to 1. From the figure 4, it can be seen that the deceleration parameter  $q$  decreases from positive value to negative value i.e., the evolution of the universe demands early deceleration and late acceleration. From equation (32) it seems that the positive part is larger in NMCG than NGCG ( $\gamma = 0$ ). Therefore, NMCG is able to describe the deceleration part of the universe in a larger range than NGCG.

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